Normalized similarity measure of 2D objects and their morphing

Hermilo Sánchez*

Department of Computer Science
Institute of Research in Applied Mathematics and Systems
National Autonomous University of Mexico
Apdo. 20-726, México, D.F., 01000. Fax: 5622-3620
E-mail: `herssan@uxmcc2.iimas.unam.mx`

Abstract

The purpose of this paper is to show that in order to obtain a normalized similarity measure, the study of some properties like *compactness*, the *work* done in object transformations and *the number of pixels* to be moved help us to establish the way of normalize a similarity measure of an appropriate set of 2D objects, including irregular objects; at the same time, it is shown that pixel representation permits break the object to get the so called *morphing*. It is also shown that orientation (by means of principal axes) of objects to be compared is crucial in the performance of the work done in their transformation.

Key words: compactness; transforming; shapes; positive pixels, morphing.

1 Introduction

It has been written about comparing, registering and recognizing of 2D objects. Sometimes there are considered contour shapes and their geometrical

*Part of this work is a section of the Doctoral Dissertation presented to the Universidad Nacional Autónoma de México properties [1], [2] and [3] to establish their similarity. Generally, in works are assumed that objects to recognize are invariant under affine formations like rotation, translation and scaling. Some authors employ tance transforms to compare objects. Arkin et. al [3] for example compar several polygons trough distance functions. One of the first formal works establish the relationship between shape and similarity degree, for 2D jects, was proposed by Bribiesca and Guzman [5]. They presented a resolution method through the so called *shape numbers*: greater resolution o 2D objects with the same shape numbers, more similar will be the compared objects.

One of the methods to represent 2D objects are through the so *picture elements*, or pixels for short. In this work, we are going to conside a 2D *object* as a binary spatial representation of a bi-dimensional scene, which every pixel takes the value of "0" or the value of "1"; as *irregula objects* we consider those objects that have no symmetry axes.

In 1997, Bribiesca and Wilson [6] intended, for the first time, to compar 2D objects through the movement of pixels, from the so called *positive* the *negative* sets of pixels, and this is what we will know as *transformatio of objects*. As *work done* in transforming objects, we consider the concep introduced by Bribiesca and Wilson [6] in the sense that while moving pixel there is a work done to translate every positive pixel to the negative Because the force involved in the translation is constant and equal to work done has a numerical value of distance. The grater distance in displacin one set into another, the more dissimilar are the objects considered.

Bribiesca and Wilson [6] did not normalize their similarity measure employed maximum correlation to orientate objects. In this paper propose the use of principal axes to orientate the objects, and provide garian algorithm to move every pixel to find the total smallest distance in transforming two objects. 2) Also we provide a method to normalize similarity measure proposed by Bribiesca and Wilson [6]; and 3) we provid a new method to make a *morphing*, through the movement of pixels, givin at the same time the degree of similarity measure.

2 Discrete compactness in 2D

Again, in 1997, Bribiesca [7] established a compactness measure of 2D object composed of pixels. This measure related contact perimeter (total length the sides that pixels get in touch) to contour perimeter. Given a 2D object composed of n pixels, let P_c be the contact perimeter. Discrete compactness, normalized in the interval [0,1], is given by the expression:

$$C_D = \frac{P_c - P_{cmin}}{P_{cmax} - P_{cmin}}, \quad (1)$$

where $P_{cmin} = n - 1$ and P_{cmax} is defined by $P_{cmax} = 2(n - n^{1/2})$.

3 Relationship between discrete compactness, work done in transforming objects and number of positive pixels

Consider twelve different objects invariant under translation (their centroids coincide), scale (same number of pixels) and rotation (aligned through their principal axes) given in figure 1. All of them are composed of 674 pixels.

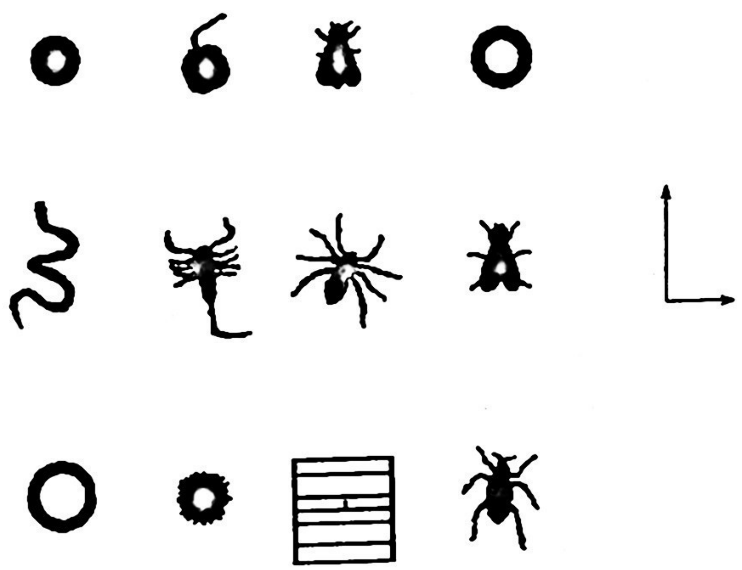

Figure 1: Twelve different 2D objects invariant under scale and rotation

To see the behaviour of compactness in terms of work done and positive pixels, let us choose the circle to be compared with the other eleven objects, because it has a high compactness (0.9954, calculated using the eq. 1) and whatever perpendicular diameters can be used as principal axes. Table and figure 2 show the work done in transforming all the objects into circle, also contains the number of positive pixels and discrete compactness of each object. As we can see, the most similar object to the circle, is circle with noise (circur), following the circle with a hair (cirpel). The most different to the circle is the frame, following the snake. Note that the frame has the smallest compactness: 0.212. Despite it does not have the maximum number of pixels to move, the work increases to 12,903.10. But, for the ring2, whose compactness is much grater than the frame (0.854), hardly the work done in transforming it to the circle is increased: 5,541.25. Another example. Note the case of the scorpion and the ring. The first has 0.32 times smaller compactness than the second. Despite the scorpion has smaller number pixels to be moved than the second, the work done in transforming it into circle is grater. So, from this sample of objects, the least compact objects are less similar to the circle than those more compact, despite sometimes these have more pixels to be moved. It is also shown that when some pixels are moved (circle with a hair) its similitude does not change so much. Also, despite the noise, the similarity with the circur is so high, showing that measure is accurate respect to the noise.

4 How to normalize the similarity measure

4.1 The set of objects is complete

As we have said last section, the objects that have small compactness tend to be more rapidly different to the circle than those with higher compactness (of course, compactness is not the only determinant feature of objects computing the similarity, remember that the number of positive pixels plays an important roll too). What is the most compact object and what is least compact object of the whole universe of shapes? If it is considered set of N objects composed of all combinations of n pixels, it is not necessary to compute all shapes, and then the work done in transforming every pair of objects until find the two most dissimilar. Clearly, the complete set objects is so huge, in comparison with the 12 objects presented in this paper.

From the results of last section, it is deduced that a way to find the most dissimilar objects, from the whole universe of N objects composed n pixels, corresponds to transform the most compact object into the object

OBJECTS	*Comm Pixels*	*Pixels* (+)	C_D	W_H
cirpel	574	100	0,90358	1702
bee	516	158	0,876296	1785,16
fly	483	191	0,816914	2083,78
beetle	433	241	0,669259	2893,49
spider	392	282	0,539259	3359,12
scorpion	379	295	0,574568	4994,21
ring	291	383	0,893951	3531,62
snake	229	445	0,751111	6920,96
frame	85	589	0,211852	12903,1
ring2	24	650	0,853827	5483,75
circur	622	52	0,873086	334,77

Table 1. Common pixels, pixels to move, discrete compactness and work done in transforming the eleven objects into the circle.

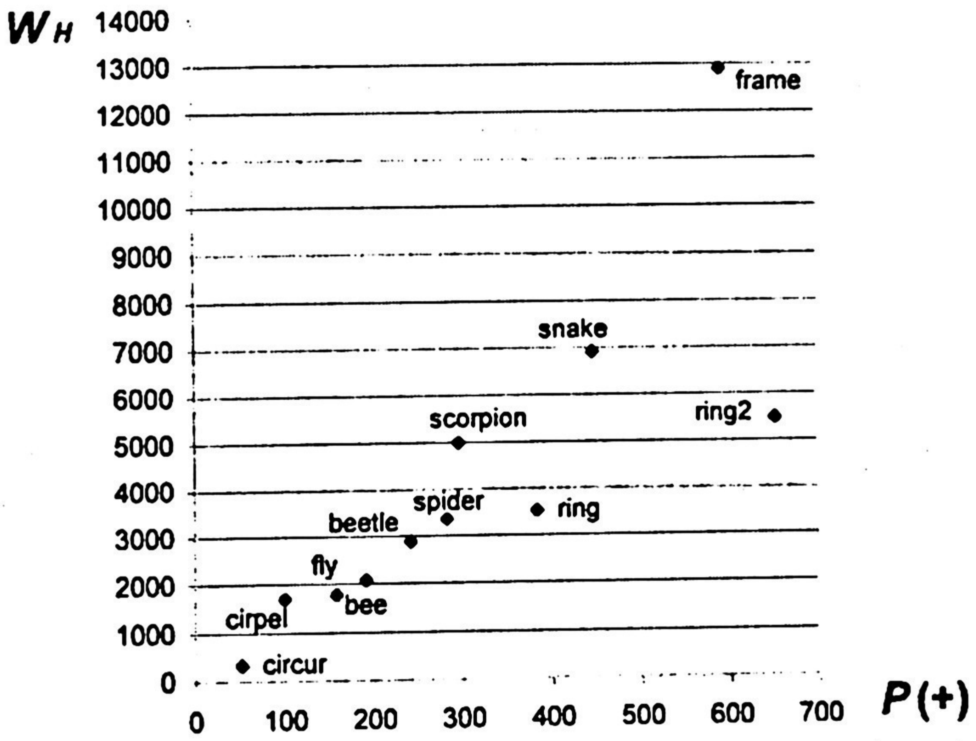

Figure 2. Work done to transform the objects as a function of positive pixels.

of smallest compactness. These objects correspond to a square of $n^{1/2}$ of side, that has the greatest compactness: 1, and a stick of length n that has the smallest compactness: 0. To know what is the work in transforming the square into the stick composed of n pixels, as big as we desire, we could obtain the work for small $n's$, and then to adjust a polynomial curve. Table shows the work done between the square and the stick, for five different values of n. Adjusting a second order polynomial extrapolation, for x = 674 pixels, the work done is approximately 454,276. What it means, is that if normalizing with this amount, the circle and the snake would be similar in 98.47% and circle and the frame in 97.16%.

Pixels	*P(+)*	*EMC*	W_H
9	6	15,54	15,28
25	20	131,14	129,6
49	42	527,06	522,82
81	72	1479,6	1471,02
121	110	3257,46	3346,14

Table 2. Work done to transform a square into a stick of 676 *pixels* each one.

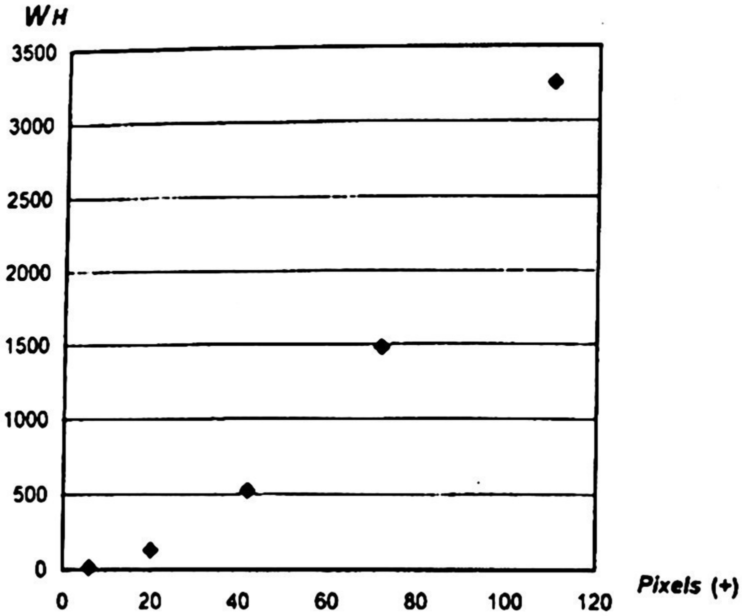

Figure 3. Work done with hungarian algorithm as a function of pixels to move, to transform a square into a stick at different resolutions.

Of course, those amounts do not correspond to the visual meaning. reason is that there are a lot of objects that have more capricious shapes that do not appear here, but there can be formed with the 674 pixels. it is more appropriate to compute the normalization in this way if having a so large representative sample of objects and with so different forms. normalize a sample like that given in section 3, is more convenient to consider another method that we propose in the next section.

4.2 The set of objects is not complete

Given a set of N objects, suppose each one is composed of n pixels. A method to normalize the similarity measure, is to look for, between the N objects, what are the two most dissimilar objects. With this amount we have maximum work done in the transformation of this N objects. Considering the 12 objects, it is observed that the two most different are the frame the circle, they correspond to the object with greatest compactness and object with the smallest compactness. Note that their compactness are near the extrems of the interval between 0 and 1. Considering the work done transforming the frame into de circle as a similarity measure of 0%, and

Transformation	Common pixels	Pixels to move	Similarity with the circle(%)
cirpel	574	100	86.81
bee	516	158	86.16
fly	483	191	83.85
beetle	433	241	77.58
spider	392	282	73.97
scorpion	379	295	61.29
ring	291	383	72.63
snake	229	445	46.36
frame	85	589	0
ring2	24	650	57.5
circur	622	52	97.4

Table 3. Number of common pixels and positive pixels to move in each transformation.

work done in transforming the circle into itself as a 100%, we can derive a normalized expression for the similarity degree between two objects:

$$S_{1,2} = (1 - \frac{W_{1,2}}{W_{m\acute{a}x}}) * 100\%, \tag{2}$$

where $W_{1,2}$ is the work done in transforming one object into another, and W_{max} is the maximum work done of the N objects transformed. Then, we can be give a table with the work normalized in transforming all the objects into the circle (Table 3). Of course, we have shown some samples from a complete set composed of 674 pixels.

5 Orientation is crucial in performing the work

From last figures it can be seen what are the most similar objects to the circle, however, following one of the properties in pattern recognition, two near points in pattern space (in our case the space is composed of two features: the work done in the transformation of objects into the circle and the number of positive pixels), correspond to points very similar. For instance, it can be said that spider is more similar to beetle than to fly, because the two first are more near in pattern space. Circle with hair is more similar to beetle than to scorpion, and so forth. To give an accurate measure in the similitude of every pair of objects, we have to transform every pair of them. Table 4 shows the values of the work, without normalizing, carried out to every pair of objects, while Table 5 shows values of normalized work.

Objects	circle	circurr	circ_pel	bee	fly	beetle	spider	ring	scorpion	ring2	sna
circle	0										
circurr	334,77	0									
circ_pel	1702	1818,89	0								
bee	1785,16	1689,01	1607,14	0							
fly	2083,78	2042,55	1124,29	723,65	0						
beetle	2893,49	2770,64	1351,85	1747,92	1762,42	0					
spider	3359,12	2195	3348,87	3212,7	3219,17	2570,85	0				
ring	3531,62	3456,6	4061,87	3487,34	3535,87	3739,13	3763,97	0			
scorpion	4994,21	4913,96	4932,39	4627,46	4062,13	3421,89	3822,41	5181,84	0		
ring2	5483,75	5410,92	5649,88	5013,2	4885,95	4818,14	4633,84	1962,94	5897,01	0	
snake	6920,96	6823,44	6292,82	7401,57	7162,41	6755,82	5212,06	6047,34	4750,16	5816,78	0
frame	12903,1	12821,7	12538,4	12537,5	12226,8	11241,5	10004,2	9947,5	10452,5	8605,07	8672

a) Work done in the object transformation, acording to Hungarian algorithm.

Objects	circle	circurr	circ_pel	bee	fly	beetle	spider	ring	scorpion	ring2	sna
circle	674										
circurr	622	674									
circ_pel	574	553	674								
bee	516	511	514	674							
fly	483	476	479	559	674						
beetle	433	240	265	468	444	674					
spider	392	391	367	369	332	351	674				
ring	291	291	235	268	239	213	204	674			
scorpion	379	371	357	353	318	352	276	204	674		
ring2	24	57	72	149	162	131	132	378	116	674	
snake	229	233	228	262	277	222	215	86	179	110	67
frame	85	78	81	65	52	69	111	52	58	46	8

b)Common pixels

Table 4. a) Work done in transformations without normalizing; b) common pixels.

Experimentally, it is found that in transforming spider into fly without aligning respect to their principal axes, making an angle of 28.6^o between their greatest axes, and with 355 no common pixels, the work employed in the transformation is $W_H = 3,642.76$. On the other hand, if the objects are aligned as their principal axes, the work done is $W_H = 3,219.17$ reducing the work in a 11.6%. Making something similar with the spider and the snake, note three different cases shown in Table 6. In the first line it is presented the smallest number of pixels of the three cases. It is found that when they are superimposed, without aligned in terms of their principal axes (see Figure

4a) the work done is $W_H = 5,685.12$, with an amount of common pixels of 229. Meanwhile aligned in terms of their principal axes (Figure 4c)) the work done is $W_H = 5,212.06$. Thus, there is an error of 8.32%. An intermediate case is given in the second line (note how, despite to have small number of common pixels between two objects, the work done was smaller).

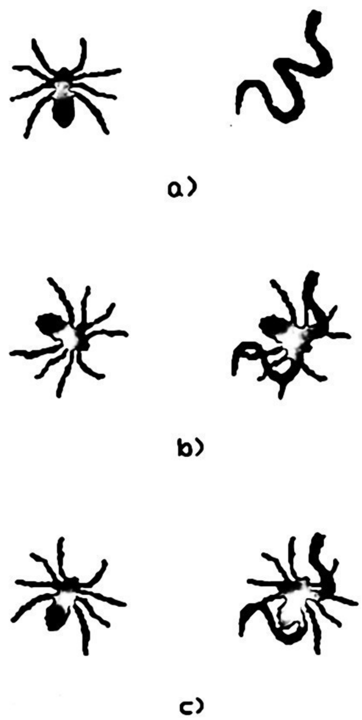

Figure 4. Transforming spider into snake; a)spider and snake b)their principal axes are at 90° between them, and the common pixels are 229; c) objects are rotation invariant with 215 common pixels

In last two lines, spider-fly, performance of transformation is better when grater axes are aligned (by 0.47° of error).

6 Morphing

According to Cohen-Or *et. al* [8] the term *morphing* is used for all methods that gradually and continuously deform one geometrical model into another. For Cheng et. al [9] morphing is a gradual change from one shape into another. Lee and Kim [10] employ Minkowski sum to transform one object into another by eliminating all redundant parts in convolution curves. Mathematically speaking, let $O^I = [O_0^I, ..., O_n^I]$ and $O^F = [O_0^F, ..., O_n^F]$ the point sets

Objects	circle	circur	cirpel	bee	fly	beetle	spider	ring	scorpion	ring2	snake
circle	100										
circur	97,41	100									
cirpel	86,81	85,90	100								
bee	86,16	86,91	87,54	100							
fly	83,85	84,17	91,29	94,39	100						
beetle	77,58	78,53	89,52	86,45	86,34	100					
spider	73,97	82,99	74,05	75,10	75,05	80,08	100				
ring	72,63	73,21	68,52	72,97	72,60	71,02	70,83	100			
scorpion	61,29	61,92	61,77	64,14	68,52	73,48	70,38	59,84	100		
ring2	57,50	58,06	56,21	61,15	62,13	62,66	64,09	84,79	54,30	100	
snake	46,36	47,12	51,23	42,64	44,49	47,64	59,61	53,13	63,19	54,92	100
frame	0	0,63	2,83	2,83	5,24	12,88	22,47	22,91	18,99	33,31	32,78

Table 5. Normalized similarity measure, from 0% to 100% for every pair of objects compared.

Object to Transform	Transformed Object Object	Angle between greatest axes	P(+)	W_H
spider	snake	169.15^o	445	5,685.12
spider	snake	90^o	459	5,598.48
spider	snake	1^o	496	5,212.06
spider	fly	28.6^o	355	3,270.25
spider	fly	0.47^o	342	3,219.17

Table 6. Number of common pixels of both objects and positive pixels to be moved in every transformation.

representing the initial and final shapes of objects, respectively. Morphing from O^I to O^F is defined by a sequence of intermediate objects as:

$$\begin{aligned} O(t) &= uO^I + tO^F \\ &= [uO^I_0 + tO^F_0, uO^I_1 + tO^F_1, ..., uO^I_n + tO^F_n] \\ &= [O_0(t), O_1(t), ..., O_n(t)] \end{aligned}$$

where $u = 1 - t$. $O_i(t)$ is the ith point in the intermediate object, formed time t. The time parameter t is normalized to the interval [0,1].

Binary representation of the objects showed here, permits transform them in order to compare them. But also, it permits break the objects to move positive pixels to conform intermediate objects. There are different orders move pixels. An order following here is by lines. We could obtain a morphing like that given by Figure 5, that shows the transformation of the fly into beetle in steps of around 10 pixels, showing 22 different discrete times.

Figure 5: Morphing of fly into beetle

Conclusions

this work, we have solved next problems: 1) similarity measure, like that given by Bribiescà and Wilson [6], has been normalized by studying compactness as an important characteristic of binary objects; 2) the measure was optimised by means of principal axes, that improves transformation of 2D objects; and 3) it has been shown that pixel representation can transform one object into another by moving positive pixels, giving a new technique to obtain a morphing. There were presented some irregular objects to apply the method of transforming 2D objects, giving good results in recognizing them.

The main disadvantage of this method is that it is slow, because every pair objects have to be compared, giving $N(N-1)/2$ comparisons. However is efficient and sound method to give the similarity degree between different 2D objects.

To improve the morphing, a future work could try to look for better functions, like that of interpolation or distance fields, giving at the same time, the similarity measure. Also, it should be better to improve the method of normalizing similarity measure without depending in the number of objects be compared.

Acknowledgments

This work was in part supported by CONACyT and UNAM. I wish to express my gratitude to Dr. Ernesto Bribiesca for his help in reviewing this work.

References

[1] Henk J.A.M. Heijmans, Alexander V. Tuzikov. Similarity and Symme Measures for Convex Shapes Using Minkowski Addition. IEEE actions on Pattern Analisys and Maching Intelligence. Vol. 20. Septiembre 1998.

[2] Van Otterloo J. Peter. A Contour-Oriented Approach to Shape Analy Prentice Hall, pp 368. 1991.

[3] Arkin, Esther M. Daniel; Chew L. Paul; Huttenlocher, Daniel P.; Klara; and Mitchel Joseph S. B. An Efficient Computable Metric Comparing Poligonal Shapes. IEEE Transactions on Pattern Analy and Machine Intelligence, vol. 13, no 3. 1991.

[4] Jain K., Fundamentals of Digital Image Processing. Prentice Hall mation and System Sciences Series. Thomas Kailath, series Editor. pp 569 .

[5] Bribiesca, Ernesto and Guzman, Adolfo. How to Describe pure form a how to Measure Differences in Shapes Using Shape Numbers. Patte Recognition 12,101-112 (1979).

[6] Bribiesca, E. and Wilson, R. G. A Measure of 2D Shape-of-Object similarity. Appl. Math. Lett. 10 6 (1997) 107-115.

[7] Bribiesca, E. Measuring 2-D shape compactness using the contact perim ter. Comput. Math. with Appl. 33: (11) 1-9 JUN 1997.

[8] D. Cohen-Or, D. Levin and A. Solomovici. Three-dimensional distan field methamorphosis, ACM Transactions on Graphics 17 (1998) 116-1

[9] Chen, Ho-Lun; Edelsbruner, Herbert and Fu, Ping. Shape space deformation. Computational Geometry. 19 (2001) 191-204.

[10] Lee, In-Kwon and Kim, Myung-Soo. Polynomial/rational approxim tion fo Minkowski sum boundary curves. Graphical models and processing. 60 (2)(1998) 136-165.